jmj

Coordinates on $\mathbb{R}^2$ (the plane)

A device to specify position in the plane

(r, θ)

Polar coordinates - another location based off circles rather than squares

Relationship between Cartesian & Polar

$x = r\cos\theta \qquad y = r\sin\theta$ & $r^2 = x^2 + y^2$ & $\tan\theta = \frac{y}{x}$

$f'(\theta) = f(\theta)\tan(\gamma - \frac{\pi}{2})$

2 ways to interpret $f'(\theta)$:

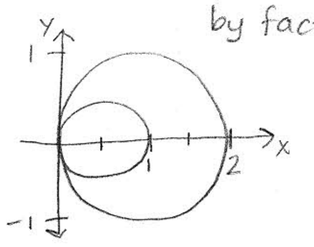
1) As the slope of $r = f(\theta)$ in the θr-plane
2) $f(\theta)\tan(\gamma - \frac{\pi}{2})$ in the xy-plane

How polar coordinates change in the xy-plane

1) How does the graph of $r = f(\theta)$ differ from $r = 2f(\theta)$?

Compare $r = \cos\theta$ & $r = 2\cos\theta$

$r = \cos\theta$	$r = 2\cos\theta$
$r^2 = r\cos\theta$	$r^2 = 2r\cos\theta$
$x^2 + y^2 = x$	$x^2 + y^2 = 2x$
$(x - \frac{1}{2})^2 + y^2 = \frac{1}{4}$	$(x-1)^2 + y^2 = 1$
radius ½	radius 1



The second is stretched by factor of 2

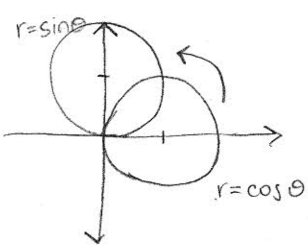
2) What is the difference between $r = f(\theta)$ & $r = f(\theta - \frac{\pi}{2})$?

Compare $r = \cos\theta$ & $r = \cos(\theta - \frac{\pi}{2})$

$r = \cos(\theta - \frac{\pi}{2}) = \cos\theta \underbrace{\cos\frac{\pi}{2}}_{=0} + \sin\theta \underbrace{\sin\frac{\pi}{2}}_{=1} = \sin\theta$

$r = \sin\theta \Rightarrow x^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$

The second is rotated 90° counterclockwise

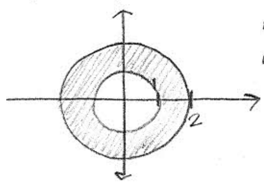


3) What is the difference between $r=f(\theta)$ and $r=-f(\theta)$?

Ex: $(-3, \pi/4) = (3, 5\pi/4)$ $\quad \theta \to \theta + \pi$ $\qquad$ So $\to$ rotation by 180°

Sketch the following regions

1) $\{(r,\theta) \mid 1 \le r \le 2\}$



"annulus"
"ring"

2) $\{(r,\theta) \mid 1 \le r \le 2,\ 0 \le \theta \le \frac{\pi}{4}\}$